ESC103 Unit 15

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November 21, 2022

Abstract

1 November 2nd Lecture

Rotation about x axis:

$$\int_a^b \pi[(f(x))^2 - (g(x))^2]dx$$

Rotation about y axis:

$$\int_c^d \pi[(F(y))^2 - (G(y))^2]dy$$

$$\int_a^b \pi \left|(f(x) - k)^2 - (g(x) - k)^2\right| dx$$

$$V = \int \pi r^2$$

Example:

$y = x^2$
$y = x$

Rotate about line y= -2
Therefore k = -2

$$V = \int_a^b \pi \left|(f(x) - k)^2 - (g(x) - k)^2\right| dx$$
$$= \int_0^1 \pi \left|(x + 2)^2 - (x^2 + 2)^2\right| dx$$
$$= \int_0^1 \pi(x^2 + 4x + 4 - x^4 - 4x^2 - 4)dx$$
$$= \int_0^1 \pi(-x^4 - 3x^2 + 4x)dx$$
$$= \frac{4}{5}\pi$$

2 5.3 Volumes by Cylindrical Shells

insrt pic 1 insrt pic 2

Area of rectangle:

$$V_i = f(x_i^*)\delta x_i$$

Therefore:

$$V \approx \sum_{i=1}^{n} 2\pi x_i^* f(x_i^*)\delta x_i$$

$$V = \int_a^b 2\pi x f(x)dx$$

What about the area between two functions wrapped around y axis (Shell Method) insrt pic 3 ask julia later

$$V = \int_a^b 2\pi x(f(x) - g(x))dx$$

Again rotation about y axis of difference of two functions in terms of y

$$V = \int_c^d 2\pi y[F(y) - G(y)]dy$$

insrt pic 4

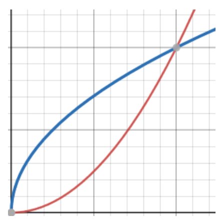


Figure 1:

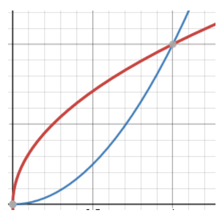


Figure 2:

Shell method for:
$y = x^2$
$y = \sqrt{x}$

$$V = \int_0^1 2\pi x[\sqrt{x} - x^2]dy$$

$$V = \int_0^1 2\pi x[x^{3/2} - x^3]dy$$

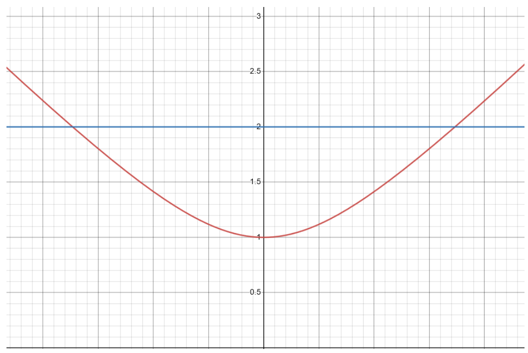
$$V = \frac{3\pi}{10}$$

Now for:
$x = y^2$
$x = \sqrt{y}$

$$V = \int_0^1 2\pi y[\sqrt{y} - y^2]dy$$

$$V = \int_0^1 2\pi y[y^{3/2} - y^3]dy$$

Example:
$y^2 - x^2 = 1$



$y = 2$ About x-axis:

$$V = 2\int_1^2 2\pi(\sqrt{y^2 - 1} - 0)ydy$$

let u $= y^2 - 1$ therefore $du = 2ydy$

$$V = 4\pi \int_0^3 \sqrt{u})\frac{du}{2}$$

$$= 2\pi[\frac{2}{3}u^{\frac{3}{2}}]_0^3$$

$$4\sqrt{3}\pi$$

Rotation about line y = k
$y = x^2$
$y = \sqrt{x}$

$$V = \int_0^1 (\sqrt{x} - x^2) \cdot (2\pi(x - k))dx$$

3 5.5 Average Value of a Function

$$a_{avg} = \frac{a_1 + a_2 + a_3 ... a_N}{N}$$

For a function:

$$f_{avg} \approx \frac{f(x_1^* + f(x_2^* + f(x_3^*)...f(x_n^*)))}{n}$$

$$\delta x = \frac{b-a}{n} => n = \frac{b-a}{\delta x}$$

$$f_{avg} \approx \frac{f(x_1^*) + f(x_2^*) + f(x_3^*)...f(x_n^*) \cdot \delta x}{b-a}$$

$$f_{avg} \approx \frac{1}{b-a} \sum_{i=1}^{n} f(x_i) \delta x_i$$

Rewriting as an integral we find:

$$f_{avg} = \frac{1}{b-a} \int_a^b f(x) dx$$

Example:

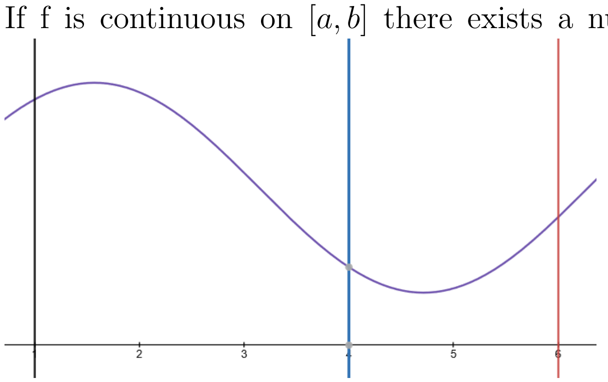
$$f(x) = x^2$$

on the inteirval $[0, 3]$

$$f_{avg} = \frac{1}{3} \int_0^3 x^2 dx = 3$$

4 Mean Value Theorem for Integrals

If f is continuous on $[a, b]$ there exists a number c in $[a, b]$ such that:



$$f(c) = f_{avg} = \frac{1}{b-a} \int_a^b f(x)dx$$

or:

$$f(c) \cdot (b-a) = \int_a^b f(x)dx$$

5 Second Mean Value Theorem for Integrals

$$\int_a^b f(x)g(x)dx = f(c) \int_a^b g(x)dx$$

f and g are continuous on [a,b] g is non-negative

Proof:

$$m \leq f(x) \leq M$$

$$mg(x) \leq f(x)g(x) \leq Mg(x)$$

$$\int_a^b mg(x)dx \leq \int_a^b f(x)dx \leq \int_a^b Mg(x)dx$$

$$m \leq \frac{\int_a^b f(x)g(x)dx}{\int_a^b g(x)dx} \leq M$$

Note: $f(c) \neq f_{avg}$ in general.

6 6: Inverse Functions